Conditional Language Modeling

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Review: Unconditional LMs

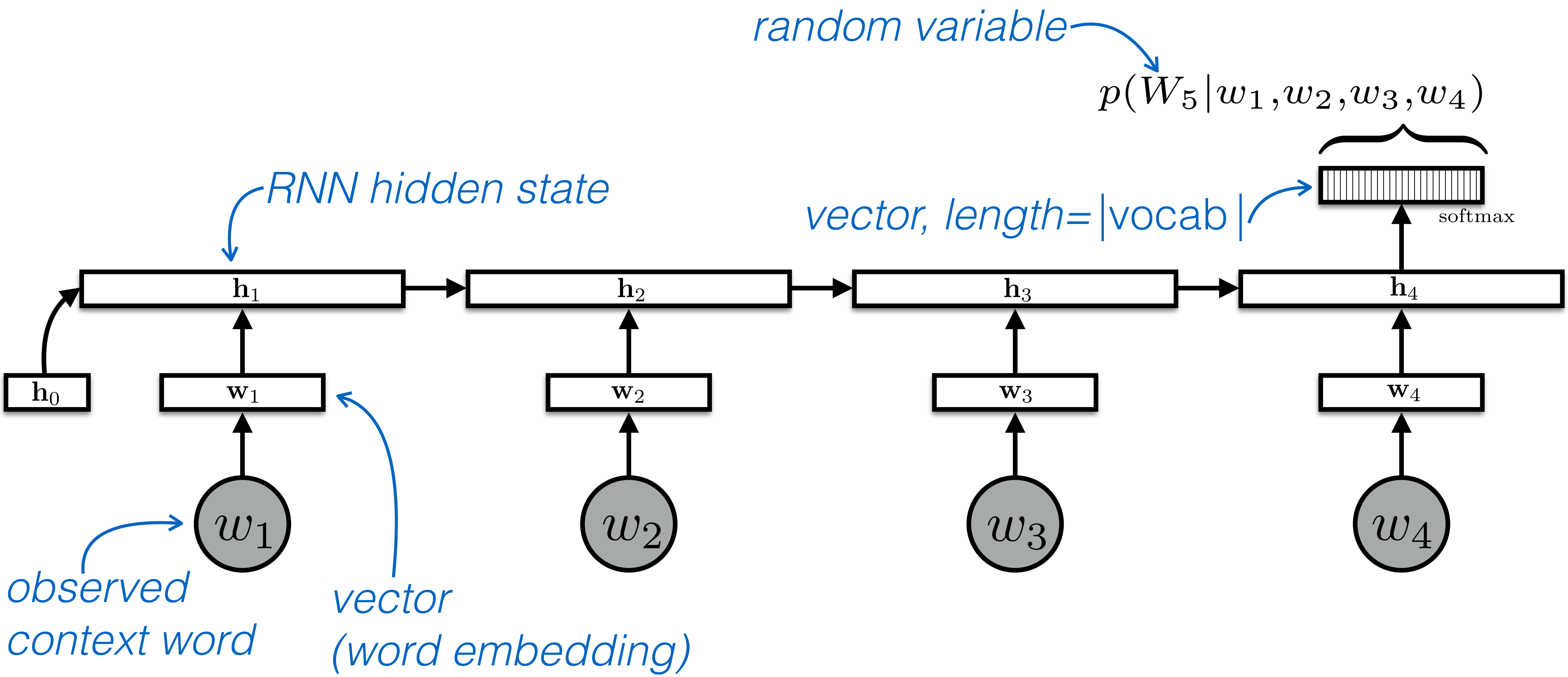
A language model assigns probabilities to sequences of words, $\boldsymbol{w} = (w_1, w_2, \ldots, w_\ell)$.

We saw that it is helpful to decompose this probability using the chain rule, as follows:

$$\begin{aligned} p(\boldsymbol{w}) &= p(w_1) \times p(w_2 \mid w_1) \times p(w_3 \mid w_1, w_2) \times \cdots \times \\ &\qquad p(w_\ell \mid w_1, \ldots, w_{\ell-1}) \\ &= \prod_{t=1}^{|\boldsymbol{w}|} p(w_t \mid w_1, \ldots, w_{t-1}) \end{aligned}$$

This reduces the language modeling problem to **modeling the probability of the next word**, given the history of preceding words.

Unconditional LMs with RNNs



Conditional LMs

A **conditional language model** assigns probabilities to sequences of words, $\boldsymbol{w} = (w_1, w_2, \ldots, w_\ell)$, given some conditioning context, $\boldsymbol{x}$.

As with unconditional models, it is again helpful to use the chain rule to decompose this probability:

$$p(\boldsymbol{w} \mid \boldsymbol{x}) = \prod_{t=1}^{\ell} p(w_t \mid \boldsymbol{x}, w_1, w_2, \ldots, w_{t-1})$$

What is the probability of the next word, given the history of previously generated words ***and*** *conditioning context* $\boldsymbol{x}$*?*

Conditional LMs

x "input"	w "**text** output"
An author	A document written by that author
A topic label	An article about that topic
{SPAM, NOT_SPAM}	An email
A sentence in French	Its English translation
A sentence in English	Its French translation
A sentence in English	Its Chinese translation
An image	A text description of the image
A document	Its summary
A document	Its translation
Meterological measurements	A weather report
Acoustic signal	Transcription of speech
Conversational history + database	Dialogue system response
A question + a document	Its answer
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this week

next week

two weeks

Data for training conditional LMs

To train conditional language models, we need *paired samples*, $\{(\boldsymbol{x}_i, \boldsymbol{w}_i)\}_{i=1}^{N}$.

Data availability varies. It's easy to think of tasks that could be solved by conditional language models, but the data just doesn't exist.

Relatively large amounts of data for:

Translation, summarisation, caption generation, speech recognition

Algorithmic challenges

We often want to find the most likely $\boldsymbol{w}$ given some $\boldsymbol{x}$. This is unfortunately generally an *intractable problem*.

$$\boldsymbol{w}^* = \arg\max_{\boldsymbol{w}} p(\boldsymbol{w} \mid \boldsymbol{x})$$

We therefore approximate it using a **beam search** or with Monte Carlo methods since $\boldsymbol{w}^{(i)} \sim p(\boldsymbol{w} \mid \boldsymbol{x})$ is often computationally easy.

Improving search/inference is an open research question.

How can we search more effectively?

Can we get guarantees that we have found the max?

Can we limit the model a bit to make search easier?

Evaluating conditional LMs

How good is our conditional language model?

These are language models, we can use **cross-entropy** or **perplexity**. *okay to implement, hard to interpret*

Task-specific evaluation. Compare the model's most likely output to human-generated expected output using a task-specific evaluation metric L.

$$\boldsymbol{w}^* = \arg\max_{\boldsymbol{w}} p(\boldsymbol{w} \mid \boldsymbol{x}) \qquad L(\boldsymbol{w}^*, \boldsymbol{w}_{ref})$$

Examples of L: BLEU, METEOR, WER, ROUGE.
easy to implement, okay to interpret

Human evaluation.
hard to implement, easy to interpret

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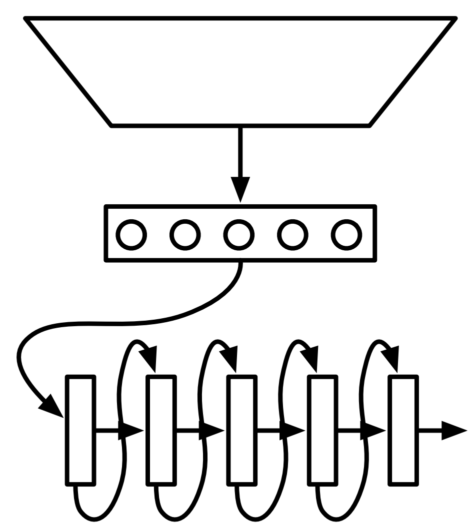
Lecture overview

The rest of this lecture will look at "encoder-decoder" models that learn a function that maps $\boldsymbol{x}$ into a fixed-size vector and then uses a language model to "decode" that vector into a sequence of words, $\boldsymbol{w}$.

$\boldsymbol{x}$ ***Kunst kann nicht gelehrt werden…***



$\boldsymbol{w}$ ***Artistry can't be taught…***

Lecture overview

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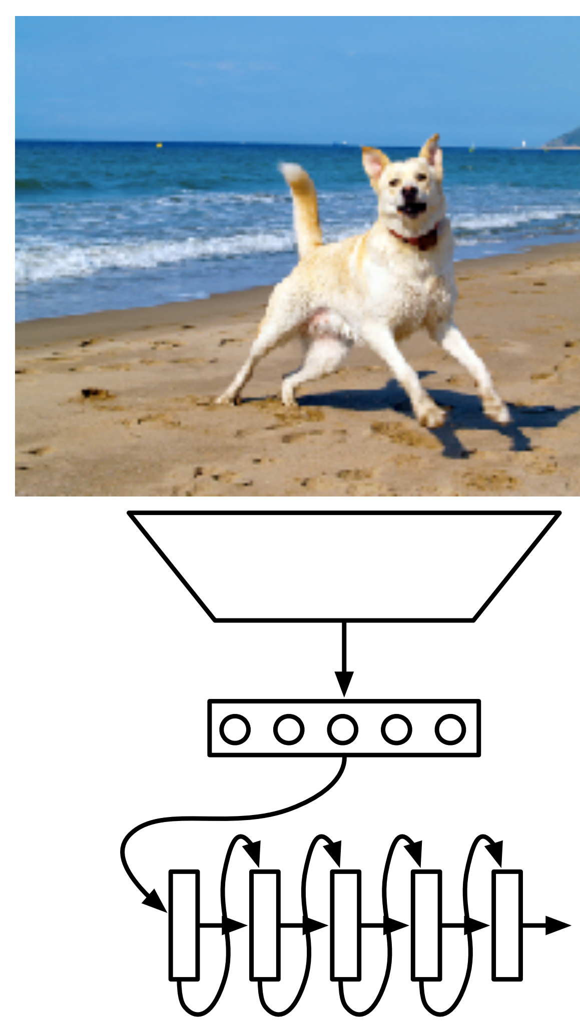
$\boldsymbol{x}$



$\boldsymbol{w}$ ***A dog is playing on the beach.***

Lecture overview

- Two questions
  - How do we encode $\boldsymbol{x}$ as a fixed-size vector, $\mathbf{c}$?
    - Problem (or at least modality) specific
    - Think about assumptions
  - How do we condition on $\mathbf{c}$ in the decoding model?
    - Less problem specific
    - We will review solution/architectures

Kalchbrenner and Blunsom 2013

Encoder

$$\mathbf{c} = \text{embed}(\boldsymbol{x})$$
$$\mathbf{s} = \mathbf{V}\mathbf{c}$$

Recurrent decoder

Recurrent connection

Embedding of w_{t-1}

Source sentence

$$\mathbf{h}_t = g(\mathbf{W}[\mathbf{h}_{t-1}; \mathbf{w}_{t-1}] + \mathbf{s} + \mathbf{b}])$$
$$\mathbf{u}_t = \mathbf{P}\mathbf{h}_t + \mathbf{b}'$$

Learnt bias

$$p(W_t \mid \boldsymbol{x}, \boldsymbol{w}_{<t}) = \text{softmax}(\mathbf{u}_t)$$

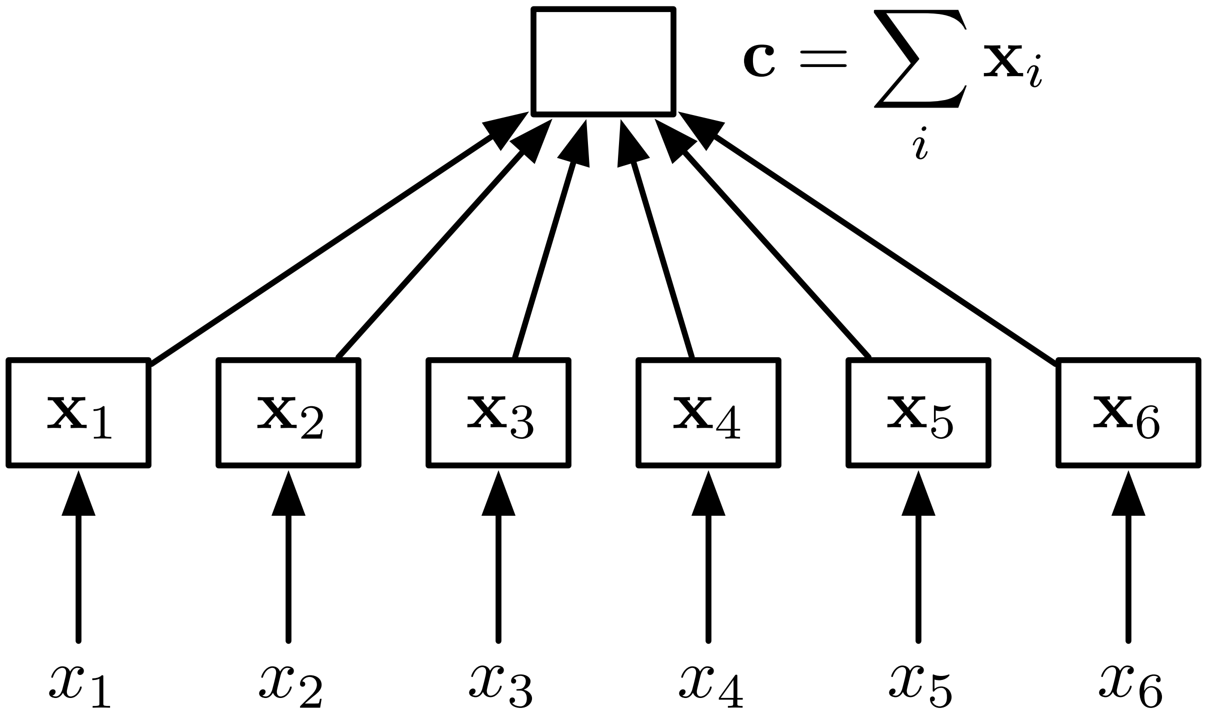
Recall unconditional RNN

$$\mathbf{h}_t = g(\mathbf{W}[\mathbf{h}_{t-1}; \mathbf{w}_{t-1}] + \mathbf{b}])$$

K&B 2013: Encoder

How should we define $\mathbf{c} = \text{embed}(\boldsymbol{x})$?

The simplest model possible:

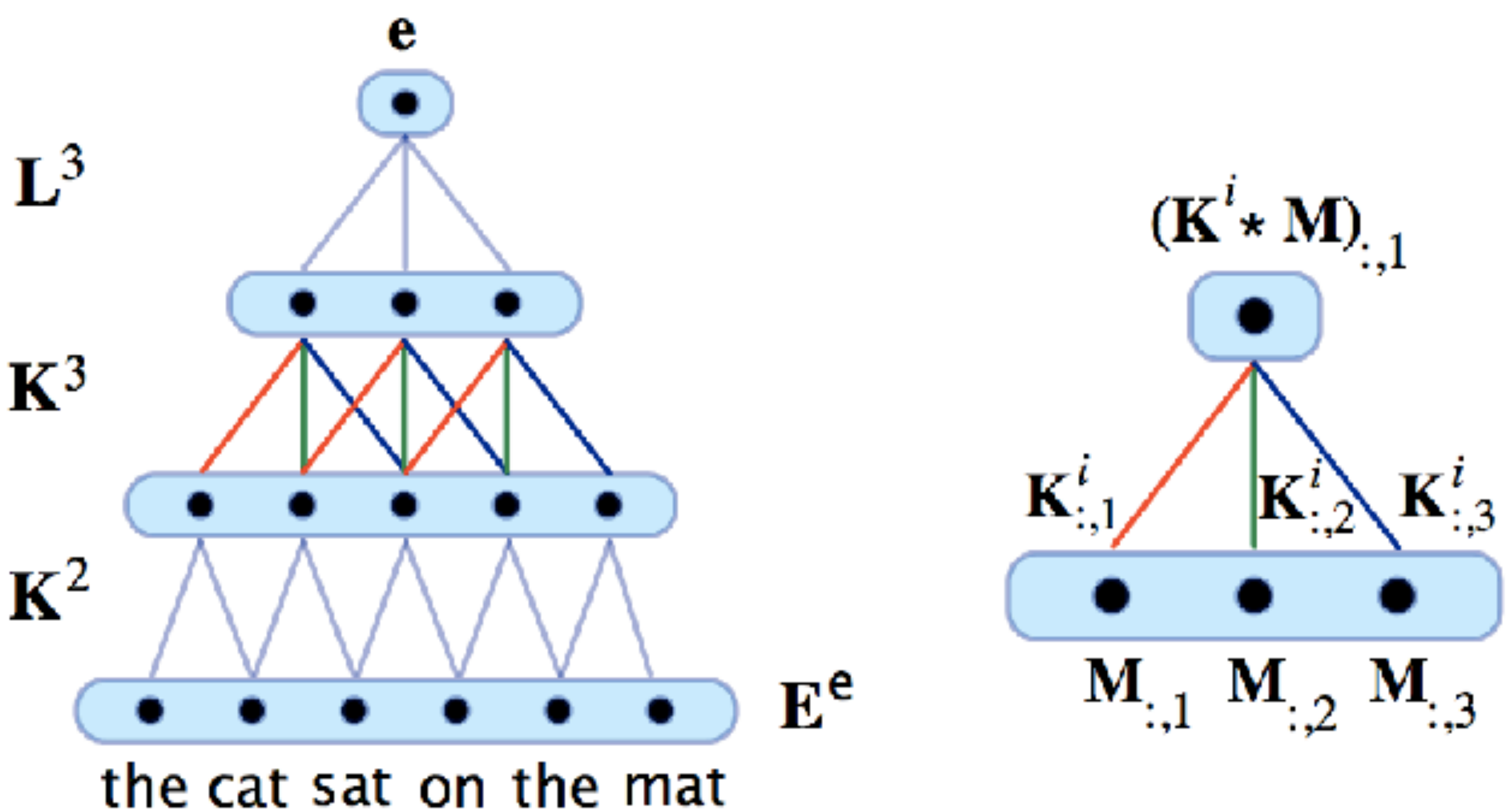


What do you think of this model?

K&B 2013: CSM Encoder

How should we define $\mathbf{c} = \text{embed}(\boldsymbol{x})$?

Convolutional sentence model (CSM)



K&B 2013: CSM Encoder

- **Good**
  - Convolutions learn interactions among features in a local context
  - By stacking them, longer range dependencies can be learnt
  - Deep ConvNets have a branching structure similar to trees, but no parser is required
- **Bad**
  - Sentences have different lengths, need different depth trees; convnets are not usually so dynamic, but see*

* Kalchbrenner et al. (2014). A convolutional neural network for modelling sentences. In *Proc. ACL*.

K&B 2013: RNN Decoder

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$$\mathbf{s} = \mathbf{V}\mathbf{c}$$

Recurrent decoder

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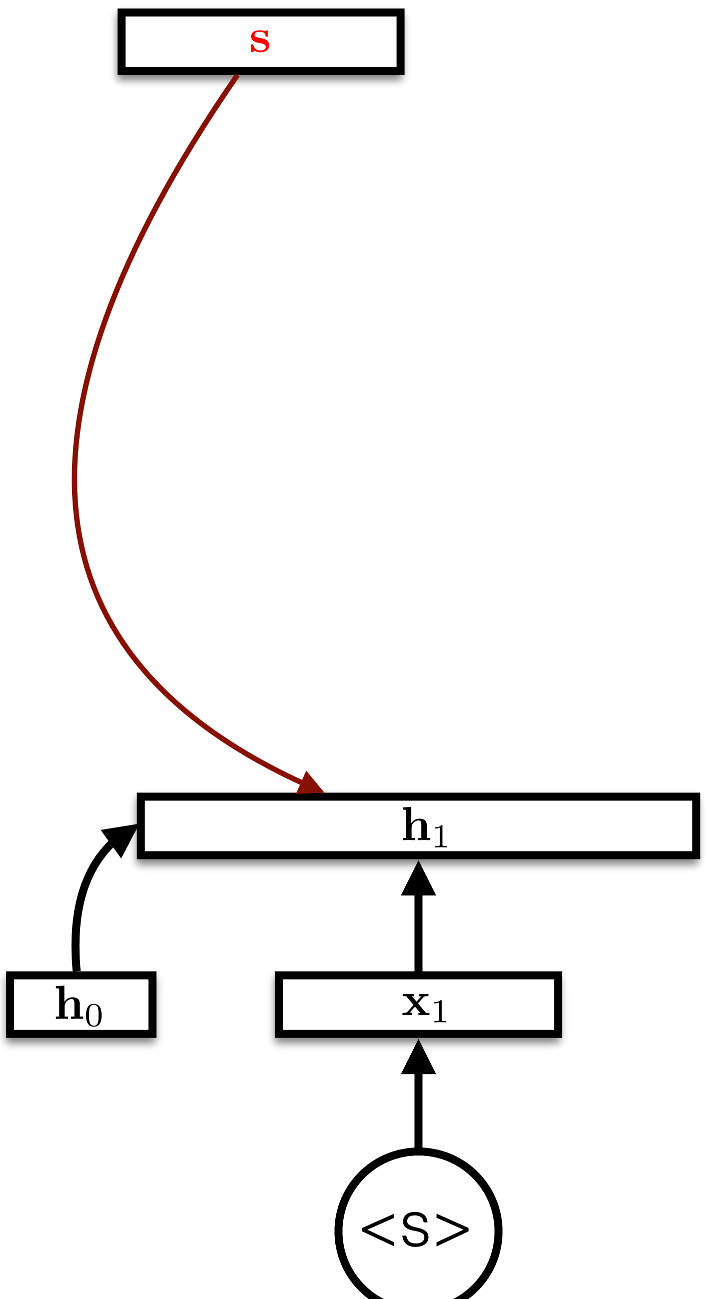
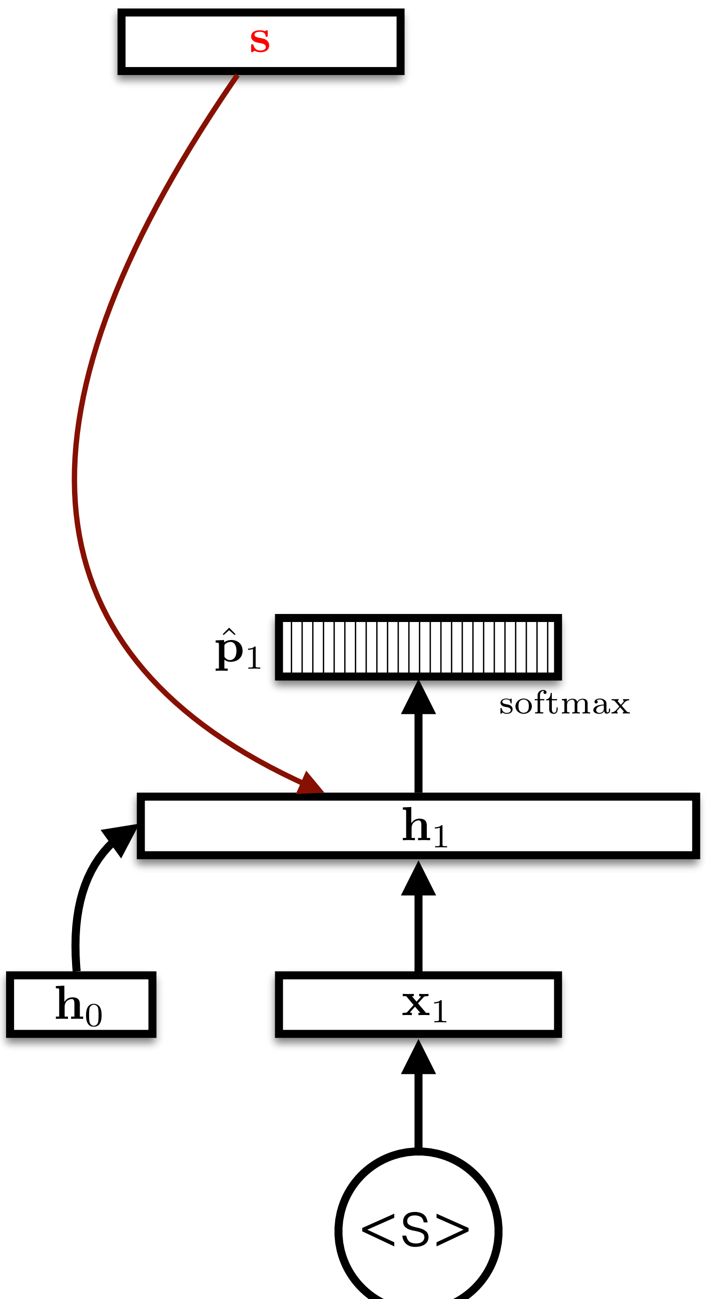
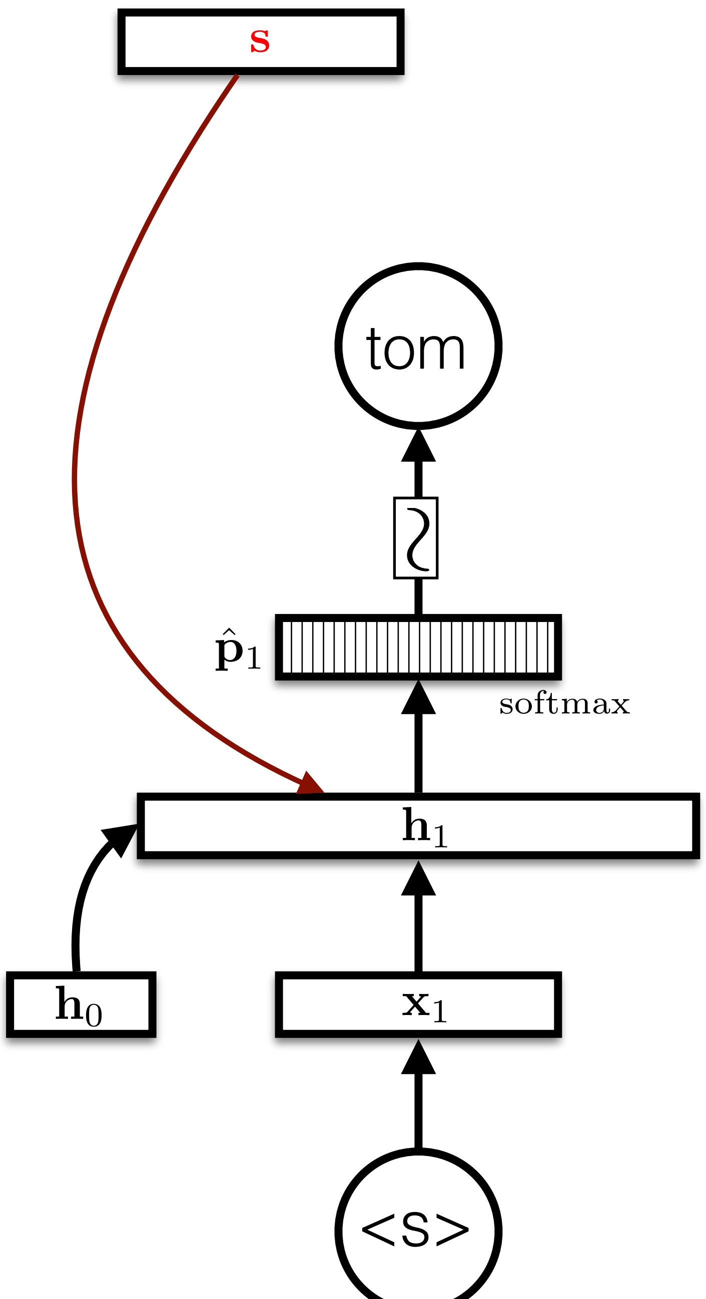


K&B 2013: RNN Decoder



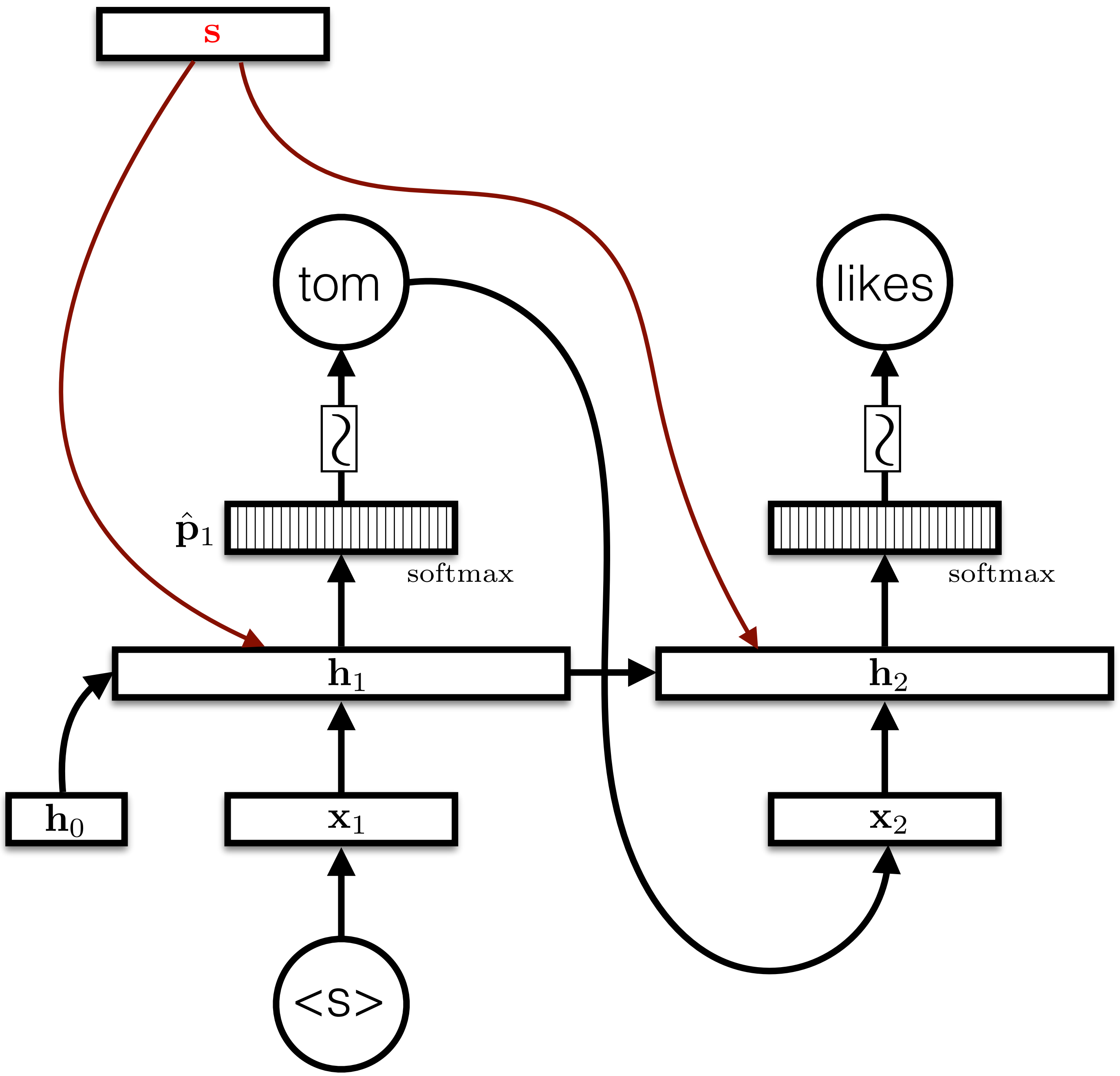
K&B 2013: RNN Decoder

$$p(tom \mid \mathbf{s}, \langle \mathbf{s} \rangle)$$



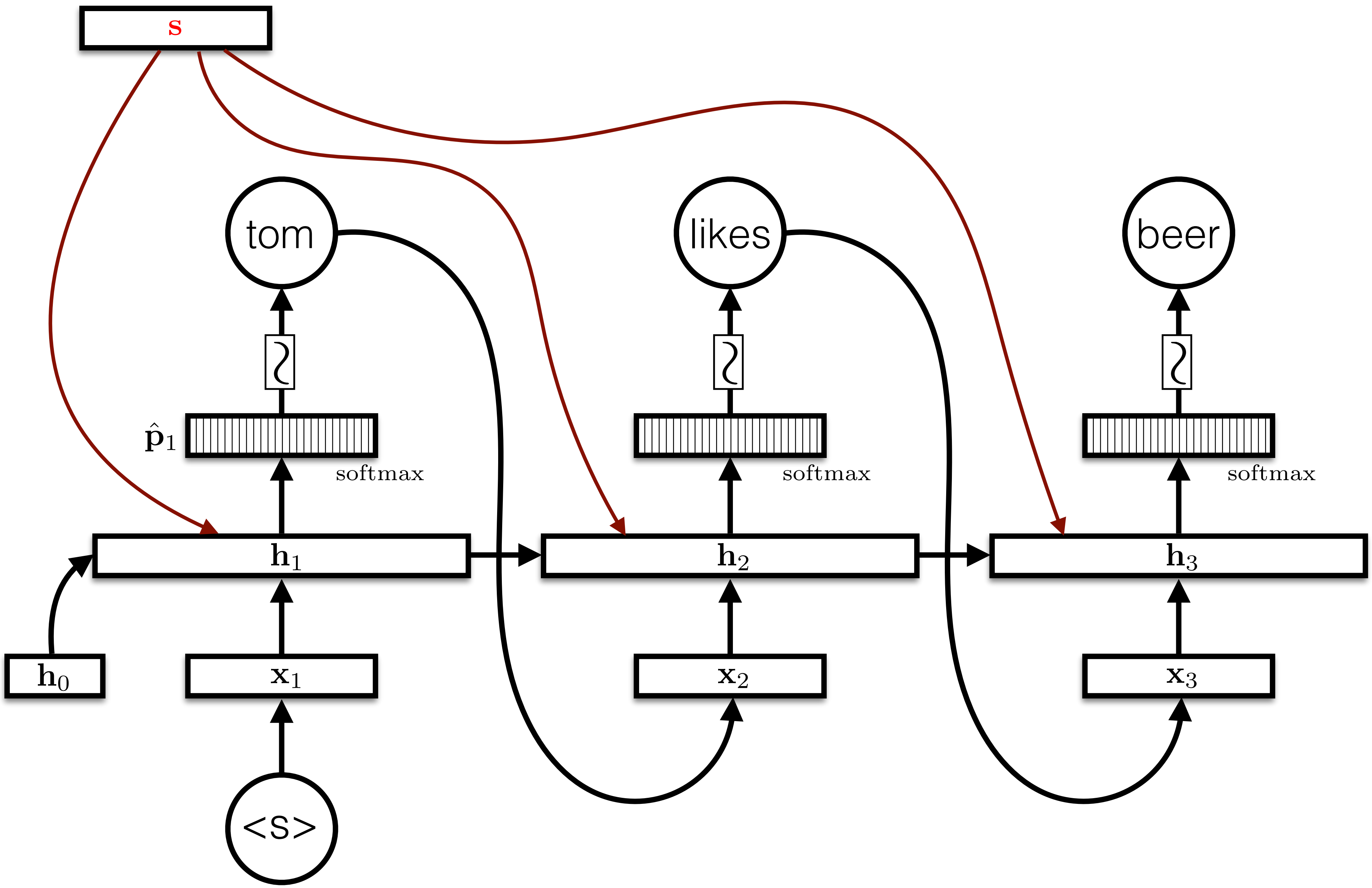
K&B 2013: RNN Decoder

$$p(tom \mid \mathbf{s}, \langle \mathbf{s} \rangle) \times p(likes \mid \mathbf{s}, \langle \mathbf{s} \rangle, tom)$$



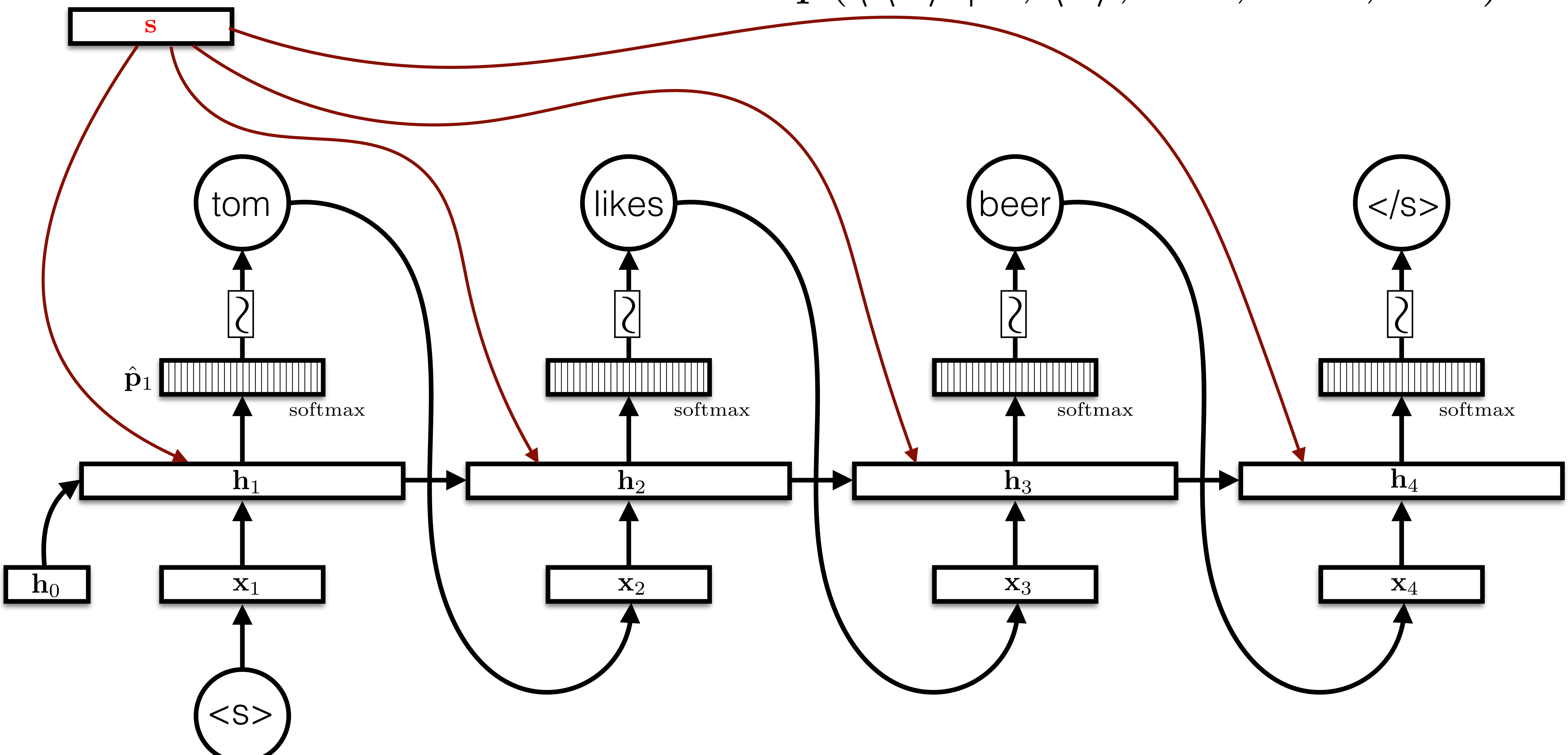
K&B 2013: RNN Decoder

$$p(tom \mid \mathbf{s}, \langle \mathbf{s} \rangle) \times p(likes \mid \mathbf{s}, \langle \mathbf{s} \rangle, tom)$$
$$\times p(beer \mid \mathbf{s}, \langle \mathbf{s} \rangle, tom, likes)$$



K&B 2013: RNN Decoder

$$p(tom \mid \mathbf{s}, \langle \mathrm{s} \rangle) \times p(likes \mid \mathbf{s}, \langle \mathrm{s} \rangle, tom) \times p(beer \mid \mathbf{s}, \langle \mathrm{s} \rangle, tom, likes) \times p(\langle \backslash \mathrm{s} \rangle \mid \mathbf{s}, \langle \mathrm{s} \rangle, tom, likes, beer)$$



Sutskever et al. (2014)

LSTM encoder

$$(\mathbf{c}_0, \mathbf{h}_0) \text{ are parameters}$$

$$(\mathbf{c}_i, \mathbf{h}_i) = \mathrm{LSTM}(x_i, \mathbf{c}_{i-1}, \mathbf{h}_{i-1})$$

The encoding is $(\mathbf{c}_\ell, \mathbf{h}_\ell)$ where $\ell = |\boldsymbol{x}|$.

LSTM decoder

$$w_0 = \langle \mathbf{s} \rangle$$

$$(\mathbf{c}_{t+\ell}, \mathbf{h}_{t+\ell}) = \mathrm{LSTM}(w_{t-1}, \mathbf{c}_{t+\ell-1}, \mathbf{h}_{t+\ell-1})$$

$$\mathbf{u}_t = \mathbf{P}\mathbf{h}_{t+\ell} + \mathbf{b}$$

$$p(W_t \mid \boldsymbol{x}, \boldsymbol{w}_{<t}) = \mathrm{softmax}(\mathbf{u}_t)$$

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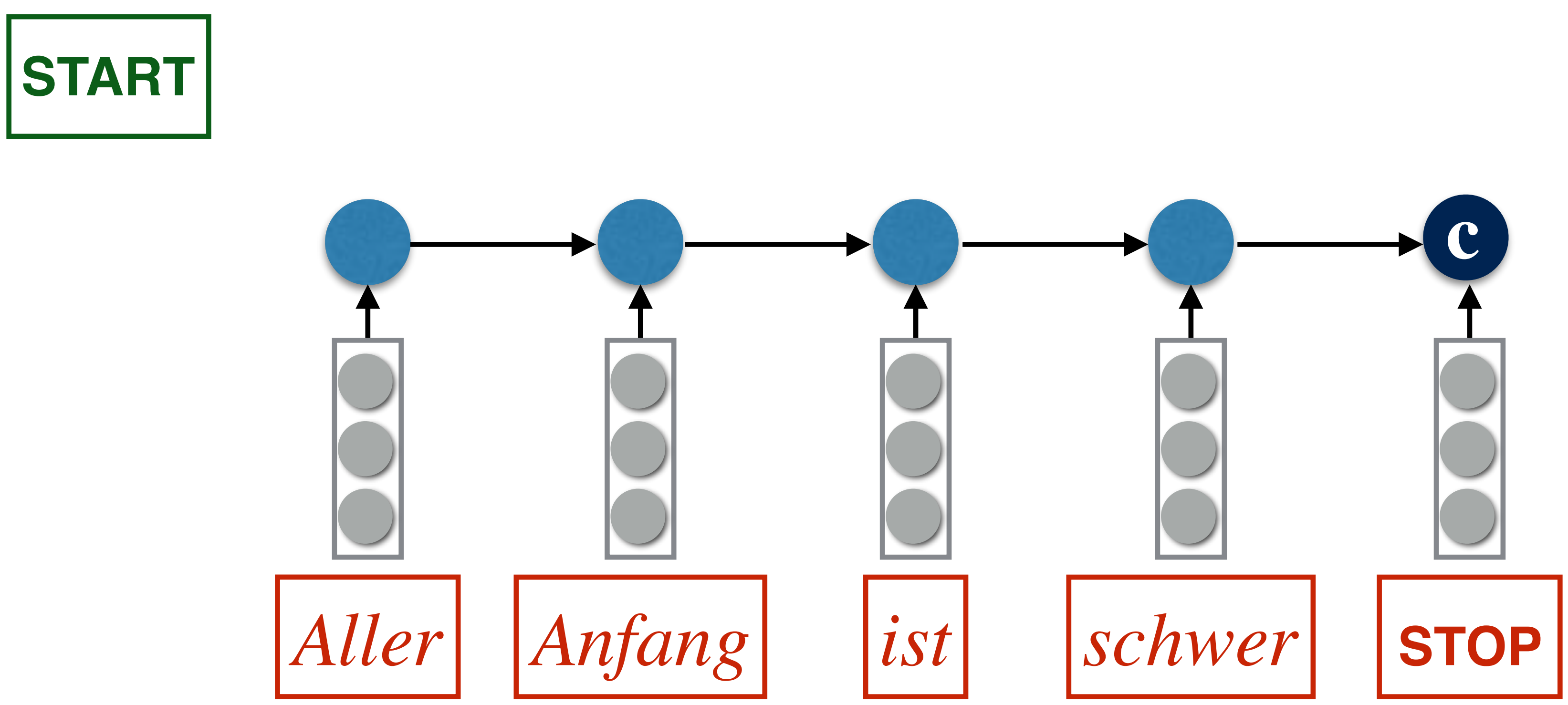

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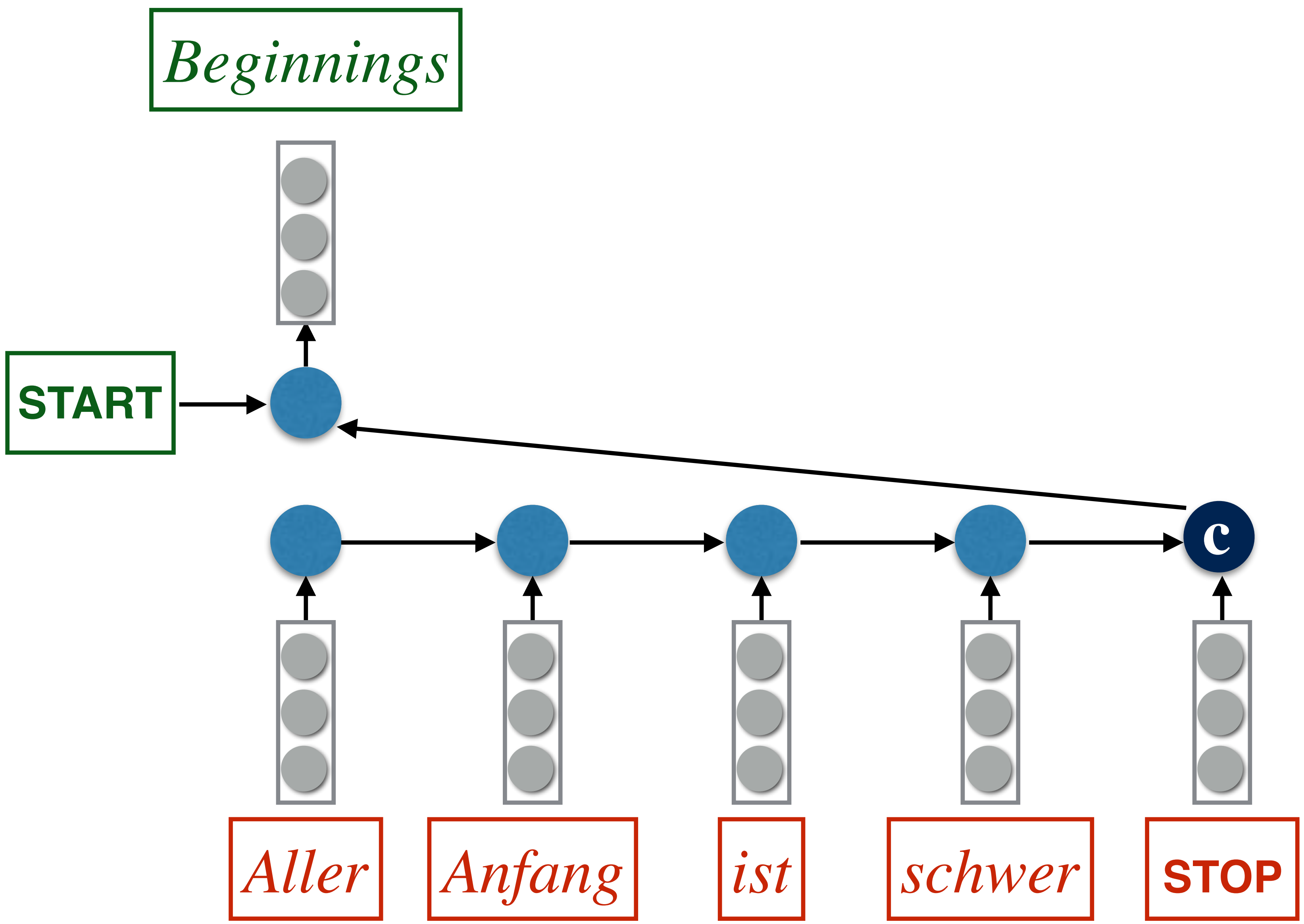

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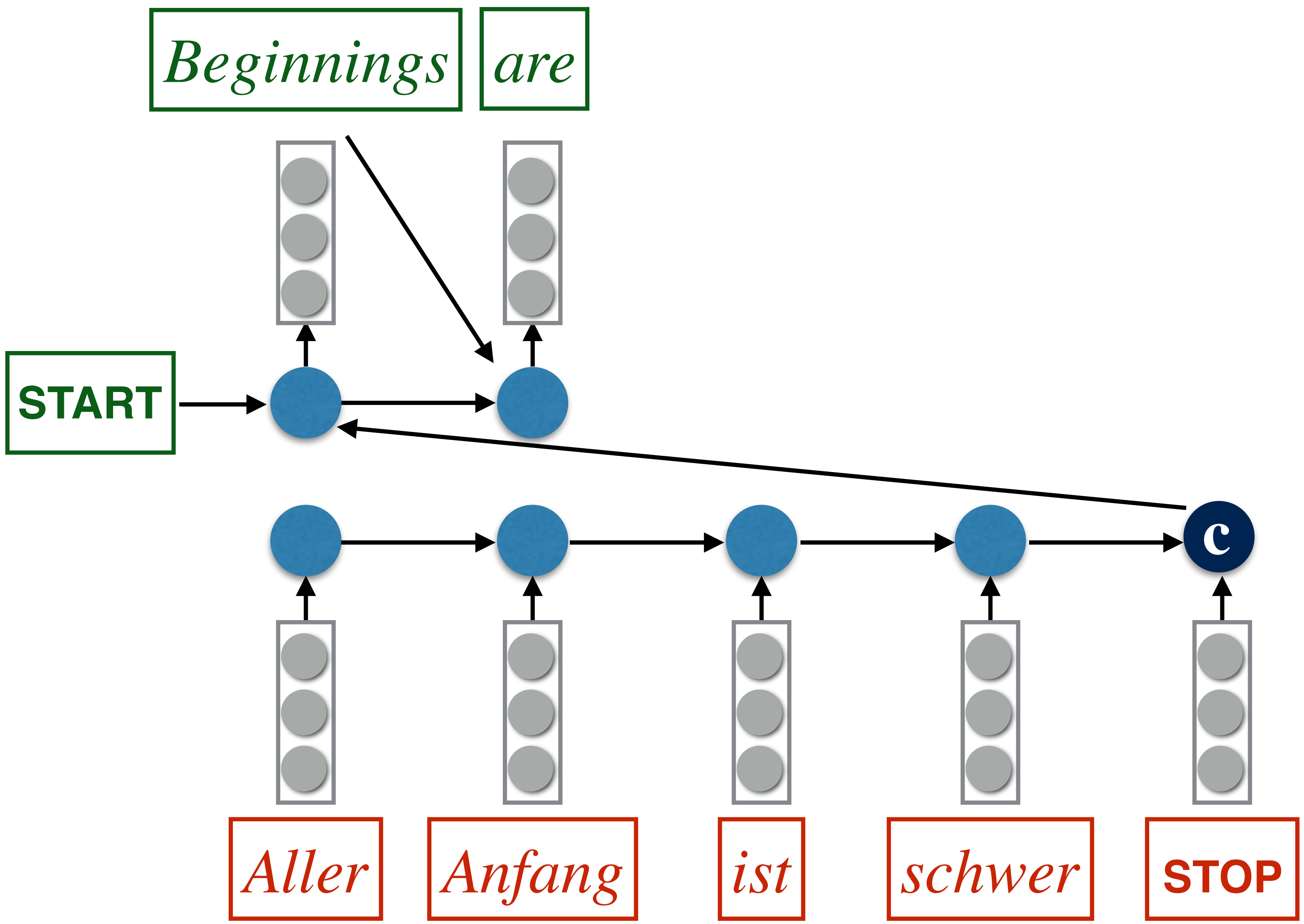

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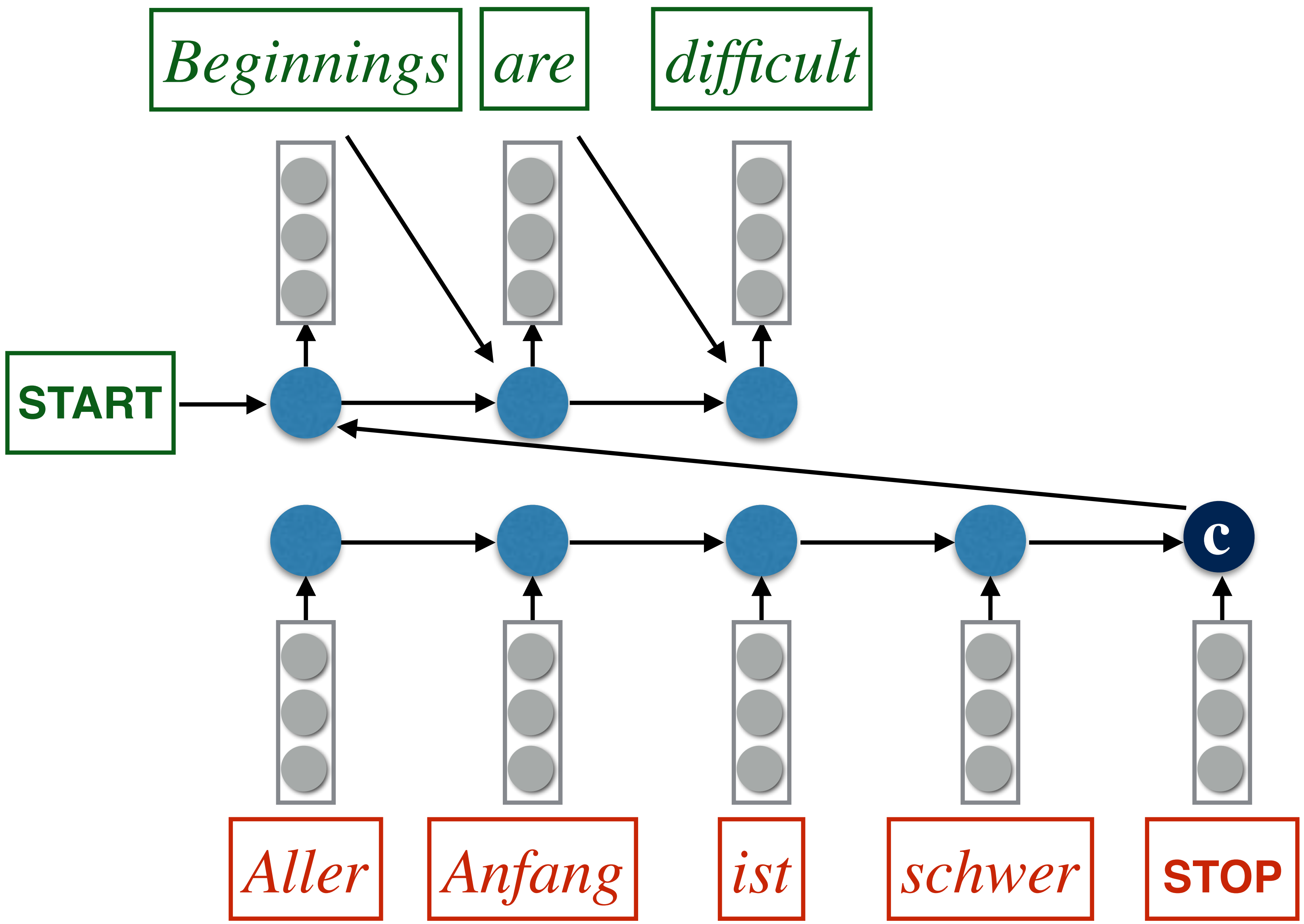

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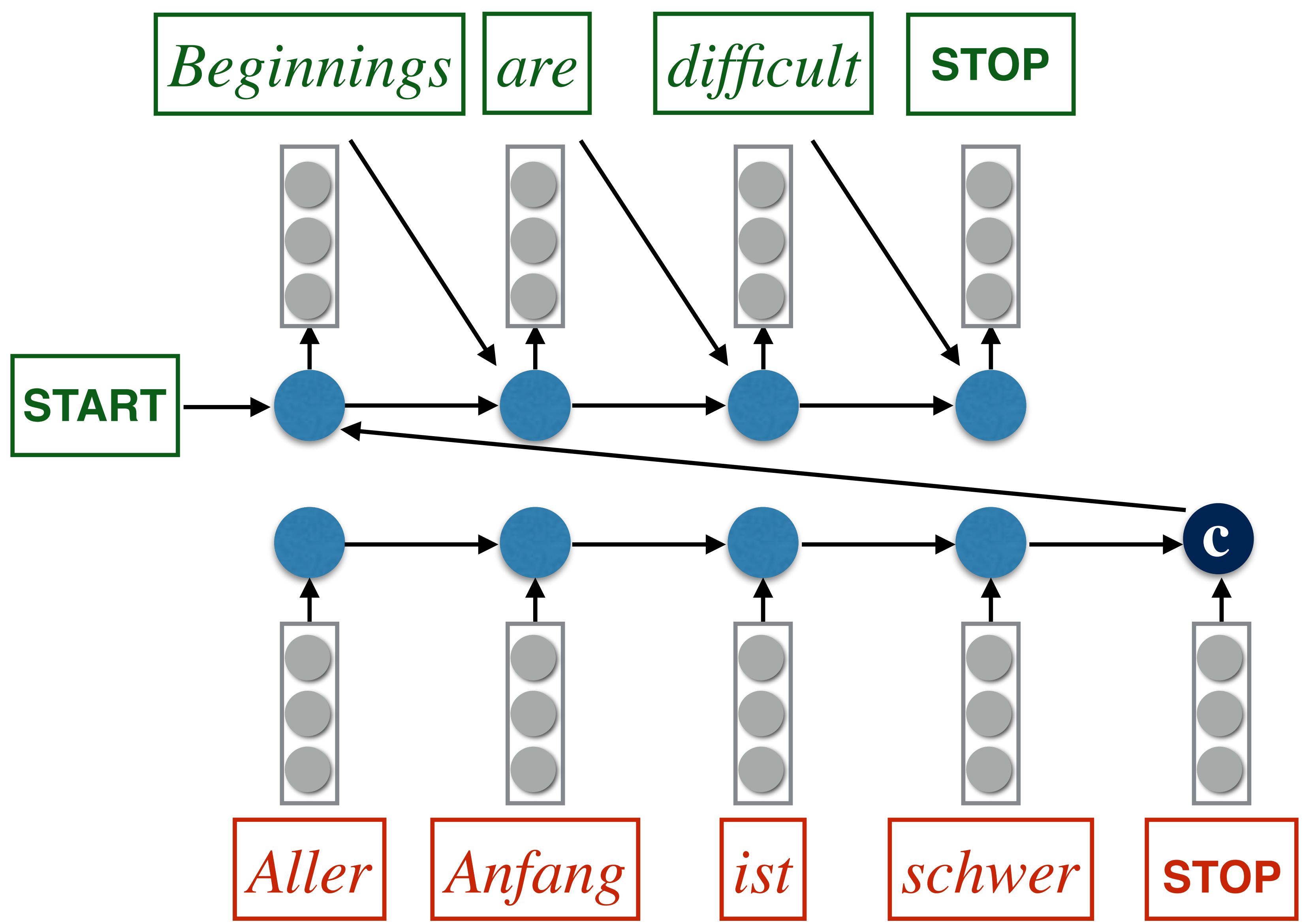
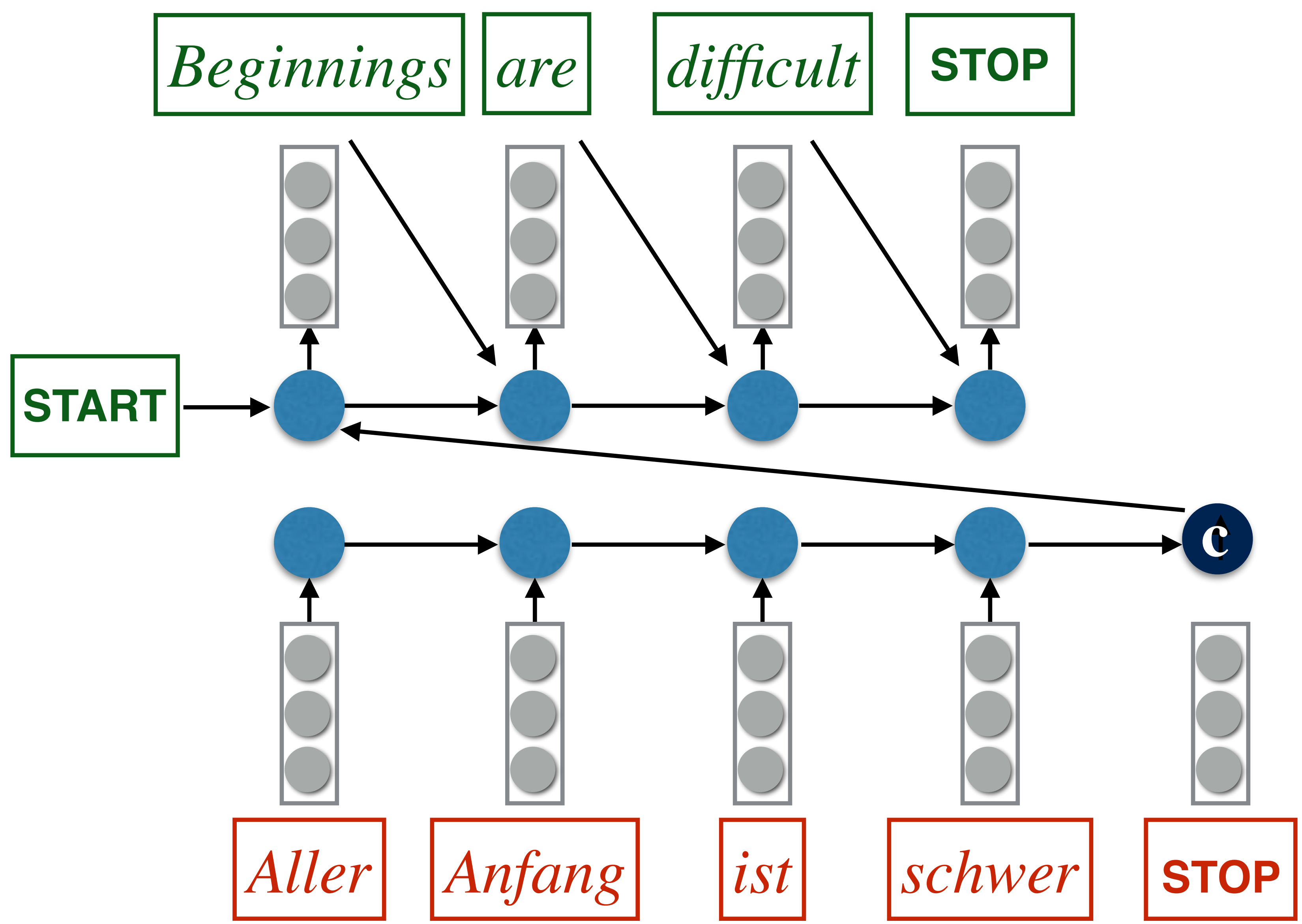


Sutskever et al. (2014)

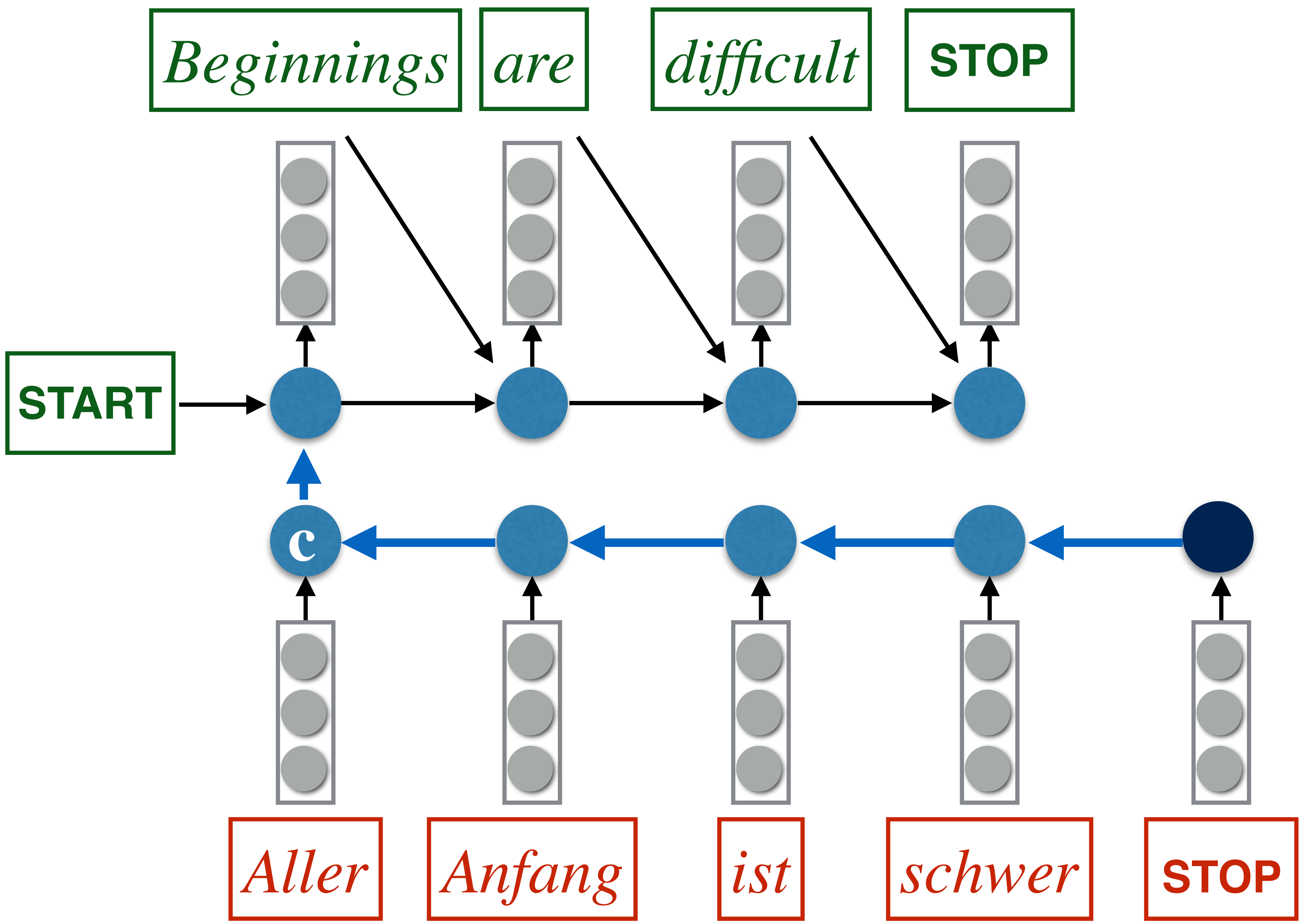
- **Good**
  - RNNs deal naturally with sequences of various lengths
  - LSTMs in principle can propagate gradients a long distance
  - Very simple architecture!
- **Bad**
  - The hidden state has to remember a lot of information! (We will return to this problem on Thursday.)

Sutskever et al. (2014): Tricks



Sutskever et al. (2014): Tricks

Read the input sequence "backwards": **+4 BLEU**



Sutskever et al. (2014): Tricks

Use an ensemble of J **independently trained** models.

Ensemble of 2 models: **+3 BLEU**

Ensemble of 5 models: **+4.5 BLEU**

Decoder:

$$(\mathbf{c}_{t+\ell}^{(j)}, \mathbf{h}_{t+\ell}^{(j)}) = \mathrm{LSTM}^{(j)}(w_{t-1}, \mathbf{c}_{t+\ell-1}^{(j)}, \mathbf{h}_{t+\ell-1}^{(j)})$$

$$\mathbf{u}_t^{(j)} = \mathbf{P}\mathbf{h}_t^{(j)} + \mathbf{b}^{(j)}$$

$$\mathbf{u}_t = \frac{1}{J}\sum_{j'=1}^{J} \mathbf{u}^{(j')}$$

$$p(W_t \mid \boldsymbol{x}, \boldsymbol{w}_{<t}) = \mathrm{softmax}(\mathbf{u}_t)$$

A word about decoding

In general, we want to find the most probable (MAP) output given the input, i.e.

$$\begin{aligned} \boldsymbol{w}^* &= \arg\max_{\boldsymbol{w}} p(\boldsymbol{w} \mid \boldsymbol{x}) \\ &= \arg\max_{\boldsymbol{w}} \sum_{t=1}^{|\boldsymbol{w}|} \log p(w_t \mid \boldsymbol{x}, \boldsymbol{w}_{<t}) \end{aligned}$$

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This is, for general RNNs, a hard problem. We therefore approximate it with a **greedy search**:

$$w_1^* = \arg\max_{w_1} p(w_1 \mid \boldsymbol{x})$$
$$w_2^* = \arg\max_{w_2} p(w_2 \mid \boldsymbol{x}, w_1^*)$$
$$\vdots$$
$$w_t^* = \arg\max_{w_2} p(w_t \mid \boldsymbol{x}, \boldsymbol{w}_{<t}^*)$$

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A word about decoding

A slightly better approximation is to use a **beam search** with beam size *b*. Key idea: keep track of top b hypothesis.

E.g., for *b*=2:

$$\boldsymbol{x} = Bier\ trinke\ ich$$

beer **drink** **I**

$\langle \mathtt{s} \rangle$ logprob=0

w_0 w_1 w_2 w_3

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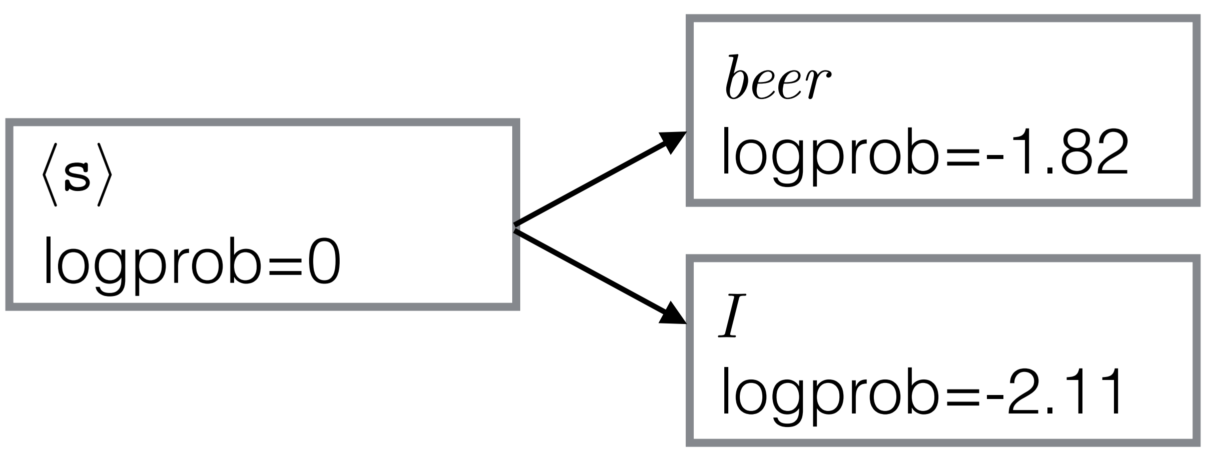

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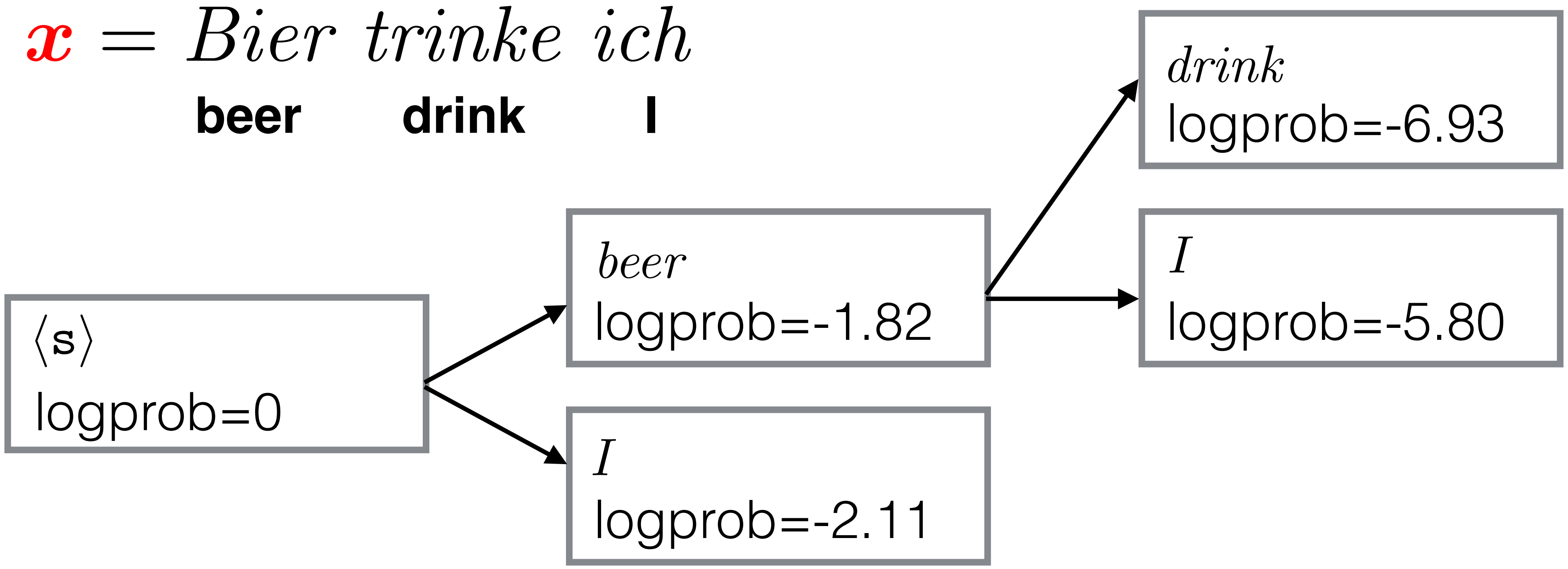

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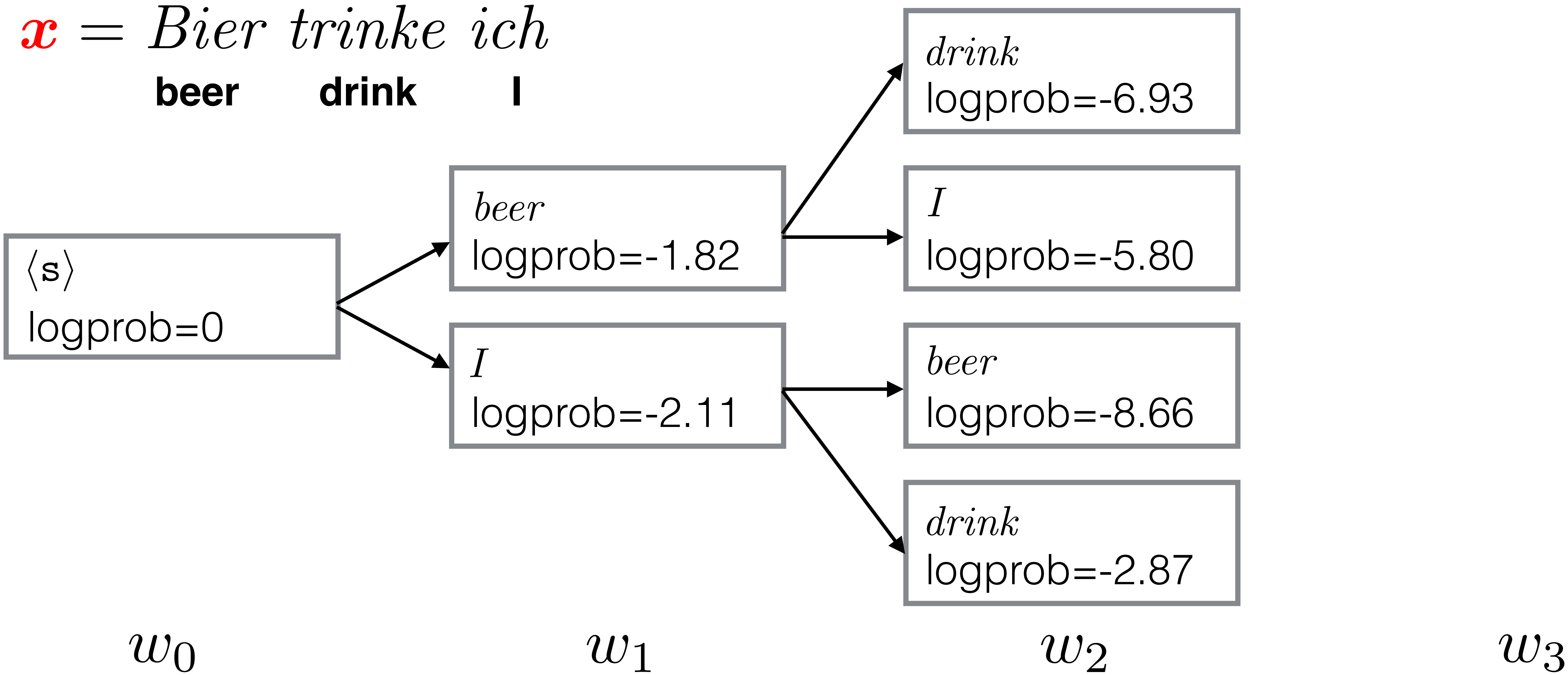

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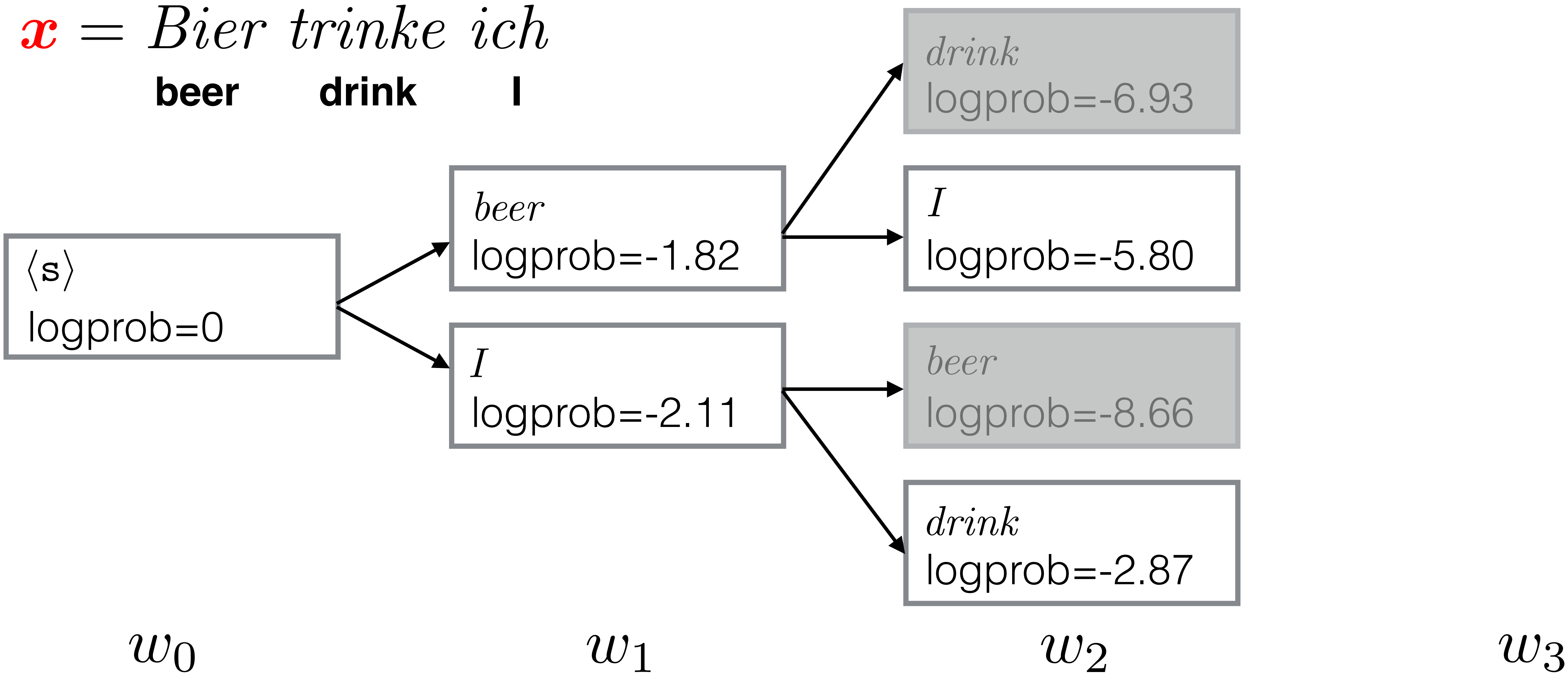

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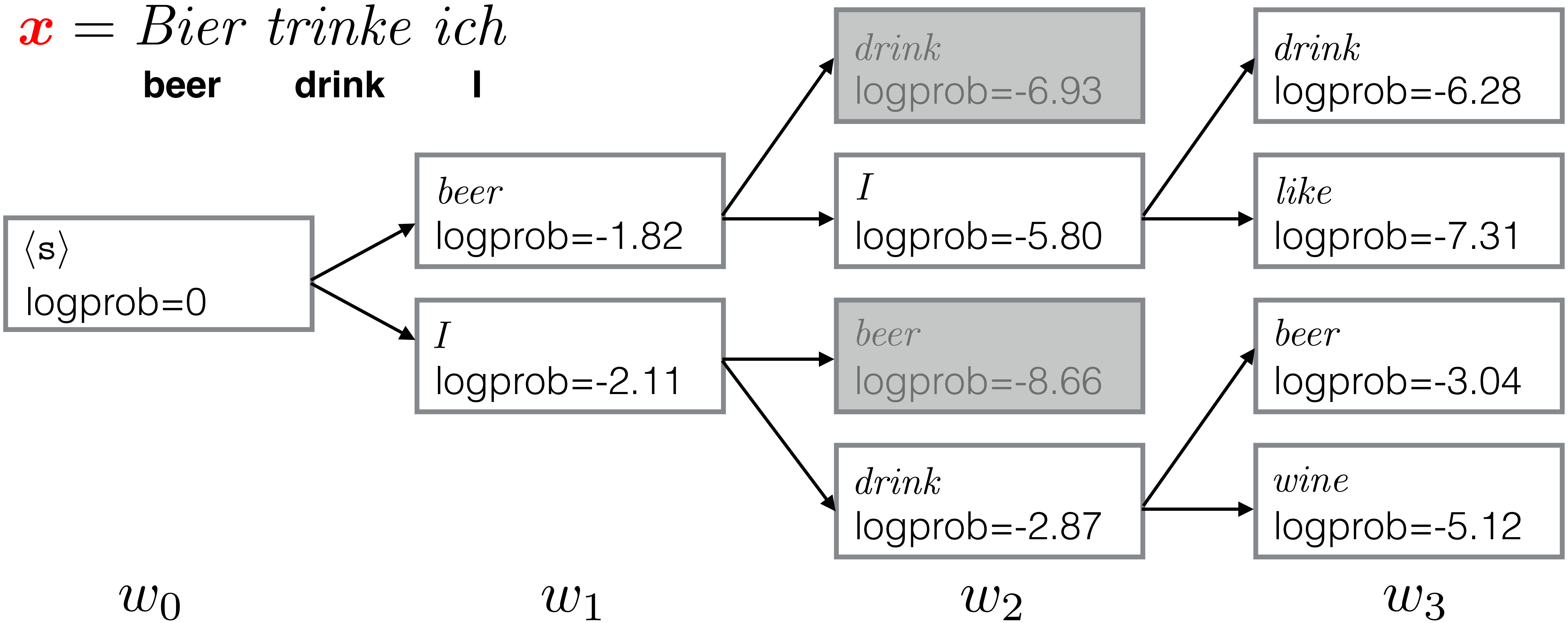

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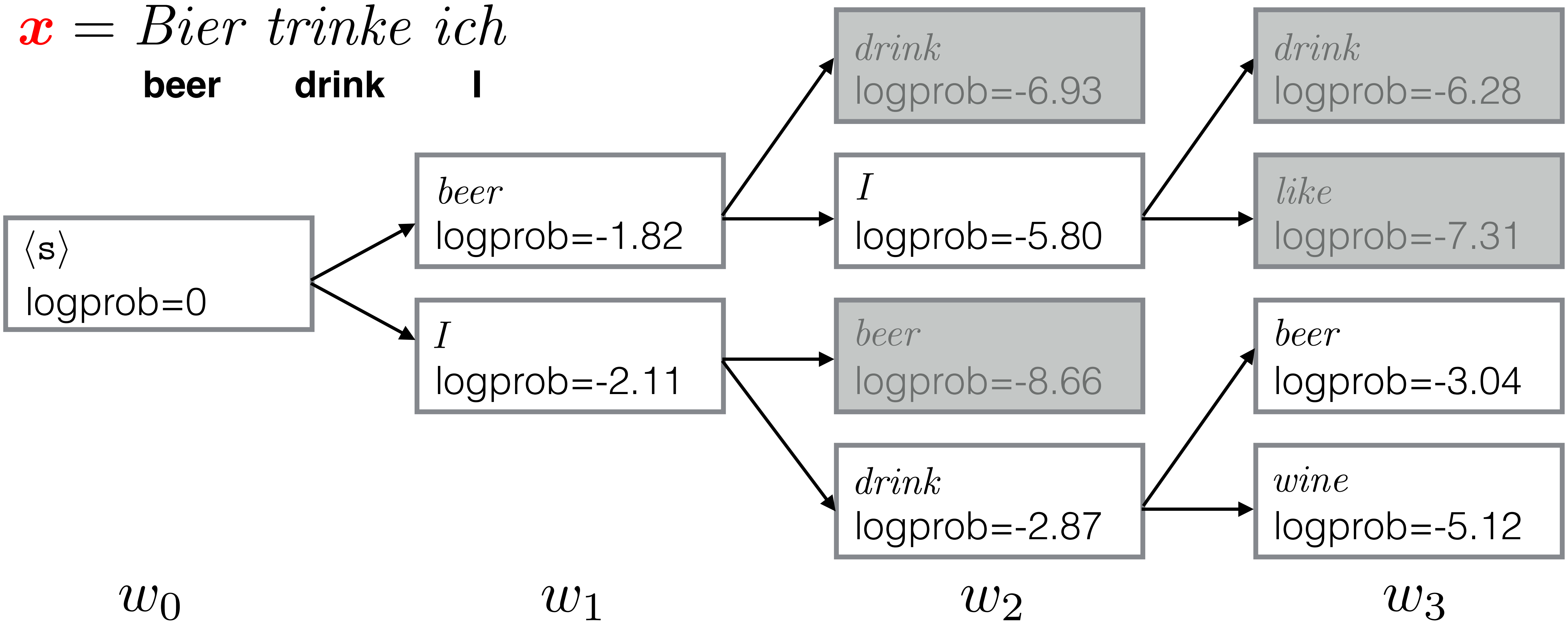

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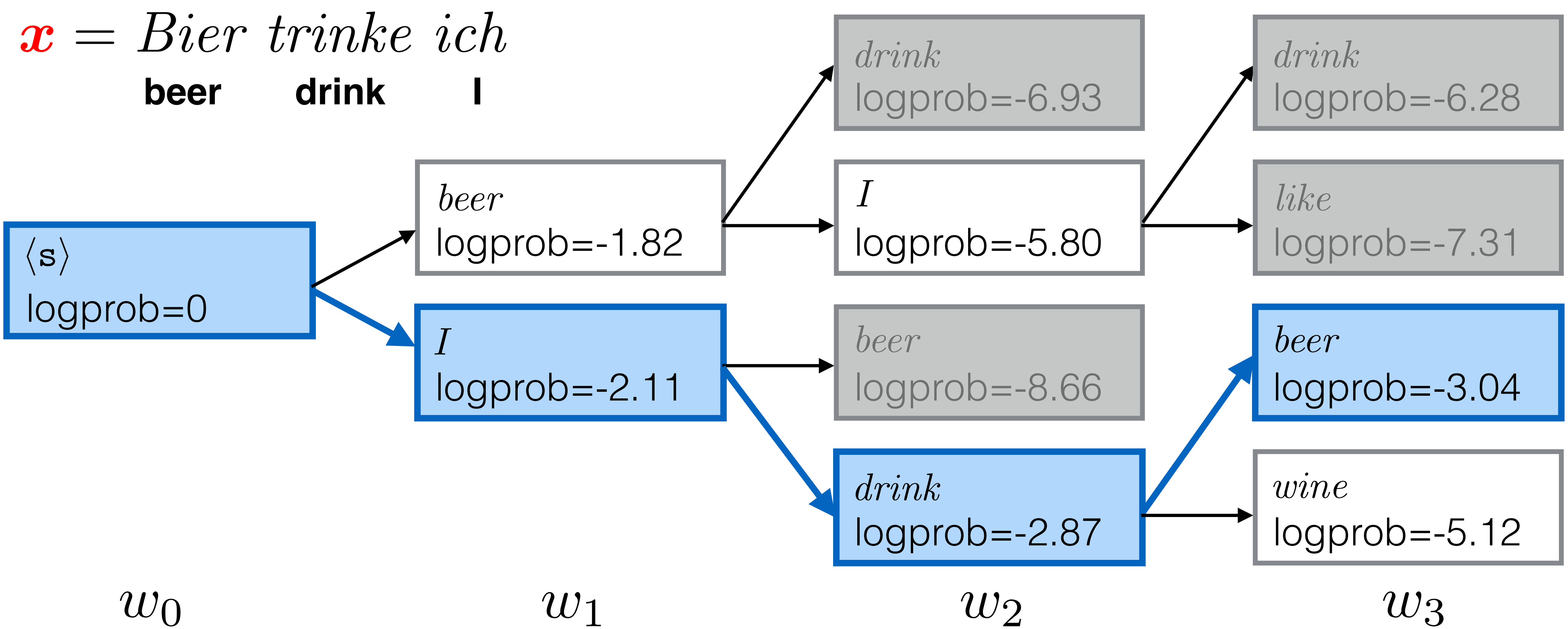

Sutskever et al. (2014): Tricks

Use beam search: **+1 BLEU**

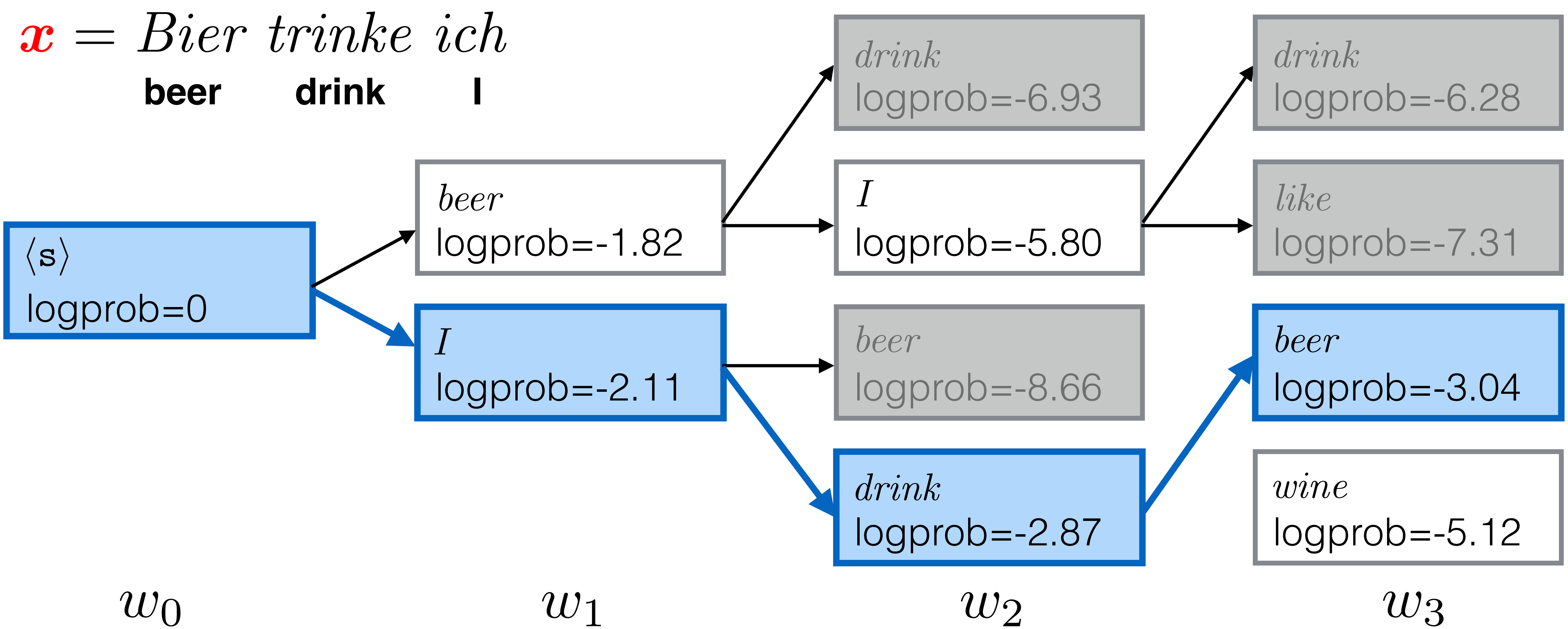

Image caption generation

- Neural networks are great for working with multiple modalities—**everything is a vector**!

- Image caption generation can therefore use the same techniques as translation modeling

- A word about data

  - Relatively few captioned images are available

  - Pre-train image embedding model using another task, like image identification (e.g., ImageNet)

Kiros et al. (2013)

- Looks a lot like Kalchbrenner and Blunsom (2013)
  - convolutional network on the input
  - n-gram language model on the output
- Innovation: **multiplicative interactions** in the decoder n-gram model

Kiros et al. (2013)

Encoder $\mathbf{x} = \text{embed}(\boldsymbol{x})$

Kiros et al. (2013)

Encoder $\mathbf{x} = \text{embed}(\boldsymbol{x})$

Unconditional *n*-gram LM: *Embedding of* w_{t-1}

$$\mathbf{h}_t = \mathbf{W}[\mathbf{w}_{t-n+1}; \mathbf{w}_{t-n+2}; \ldots; \mathbf{w}_{t-1}]$$

$$\mathbf{u}_t = \mathbf{P}\mathbf{h}_t + \mathbf{b}$$

$$p(W_t \mid \boldsymbol{x}, \boldsymbol{w}_{t-n+1}^{t-1}) = \text{softmax}(\mathbf{u}_t)$$

Kiros et al. (2013)

Encoder $\mathbf{x} = \text{embed}(\boldsymbol{x})$

Simple conditional *n*-gram LM:

$$\mathbf{h}_t = \mathbf{W}[\mathbf{w}_{t-n+1}; \mathbf{w}_{t-n+2}; \ldots; \mathbf{w}_{t-1}] + \mathbf{C}\mathbf{x}$$

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Multiplicative *n*-gram LM:

$$w_i = r_{i,w}$$

Kiros et al. (2013)

Encoder $\mathbf{x} = \text{embed}(\boldsymbol{x})$

Simple conditional *n*-gram LM:

$$\mathbf{h}_t = \mathbf{W}[\mathbf{w}_{t-n+1}; \mathbf{w}_{t-n+2}; \ldots; \mathbf{w}_{t-1}] + \mathbf{Cx}$$

$$\mathbf{u}_t = \mathbf{P}\mathbf{h}_t + \mathbf{b}$$

$$p(W_t \mid \boldsymbol{x}, \boldsymbol{w}_{t-n+1}^{t-1}) = \text{softmax}(\mathbf{u}_t)$$

Multiplicative *n*-gram LM:

$$w_i = r_{i,w}$$

$$w_i = r_{i,j,w} x_j$$

Kiros et al. (2013)

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how big is this tensor?

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Kiros et al. (2013)

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Kiros et al. (2013)

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Simple conditional *n*-gram LM:

$$\mathbf{h}_t = \mathbf{W}[\mathbf{w}_{t-n+1}; \mathbf{w}_{t-n+2}; \ldots; \mathbf{w}_{t-1}] + \mathbf{C}\mathbf{x}$$

$$\mathbf{u}_t = \mathbf{P}\mathbf{h}_t + \mathbf{b}$$

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$$\sout{w_i = r_{i,w}}$$

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Kiros et al. (2013)

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what's the intuition here?

Kiros et al. (2013)

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how big is this tensor?

Kiros et al. (2013)

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Simple conditional *n*-gram LM:

$$\mathbf{h}_t = \mathbf{W}[\mathbf{w}_{t-n+1}; \mathbf{w}_{t-n+2}; \ldots; \mathbf{w}_{t-1}] + \mathbf{C}\mathbf{x}$$

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Multiplicative *n*-gram LM:

~~$w_i = r_{i,w}$~~

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Kiros et al. (2013)

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Multiplicative *n*-gram LM:

$$w_i = r_{i,w}$$

$$w_i = r_{i,j,w} x_j$$

$$w_i = u_{w,i} v_{i,j} \qquad (\mathbf{U} \in \mathbb{R}^{|V| \times d}, \quad \mathbf{V} \in \mathbb{R}^{d \times k})$$

$$\mathbf{r}_t = \mathbf{W}[\mathbf{w}_{t-n+1}; \mathbf{w}_{t-n+2}; \ldots; \mathbf{w}_{t-1}] + \mathbf{Cx}$$

Kiros et al. (2013)

Encoder $\mathbf{x} = \text{embed}(\boldsymbol{x})$

Simple conditional *n*-gram LM:

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$$\mathbf{r}_t = \mathbf{W}[\mathbf{w}_{t-n+1}; \mathbf{w}_{t-n+2}; \ldots; \mathbf{w}_{t-1}] + \mathbf{Cx}$$

$$\mathbf{h}_t = (\mathbf{W}^{fr}\mathbf{r}_t) \odot (\mathbf{W}^{fx}\mathbf{x})$$

$$\mathbf{u}_t = \mathbf{P}\mathbf{h}_t + \mathbf{b}$$

$$p(W_t \mid \boldsymbol{x}, \boldsymbol{w}_{<t}) = \text{softmax}(\mathbf{u}_t)$$

Kiros et al. (2013)

- Two take-home messages:
  - Feed-forward n-gram models can be used in place of RNNs in conditional models
  - Modeling interactions between input modalities holds a lot of promise
    - Although MLP-type models can approximate higher order tensors, multiplicative models appear to make learning interactions easier

Questions?